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Helium-like ion forbidden line emission, and solar active regions

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A theory has been developed for interpreting the intensity of the $1s^2\ {}^1S$–$1s2s\ {}^3S$ forbidden line in helium-like ions in terms of electron density at the source. In a recent soft X-ray rocket experiment, this transition has been observed simultaneously from C v, N vi and O vii. New values for the forbidden transition probabilities are derived in a semi-empirical manner using this data. The new values lead to derived solar densities in active regions of between 10^{11} and 10^{13} cm^{-3}.

1. Introduction

Following the identification (Gabriel & Jordan 1969*a*) of a sequence of intense lines in the solar soft X-ray spectrum as the forbidden magnetic dipole transitions $1s^2\,{}^1S$–$1s2s\,{}^3S$ in helium-like ions, Griem (1969) estimated values for the transition probabilities based upon Dirac theory calculations in hydrogen. Using these values, it was possible to construct a theory for the relative intensities of the lines emitted in the Sun, in terms of electron density in the emitting regions (Gabriel & Jordan 1969*b*, hereafter referred to as Paper 1). Since then, there have been a number of new solar observations, as well as some discussions regarding the validity of the transition probabilities used. The object of the present paper is to review the current position regarding such density measurements, in the light of the new data available.

2. Principle of the analysis

For details of the theory for interpreting the relative intensities, the reader is referred to Paper 1. The intensity ratio R of the forbidden line $1s^2\,{}^1S$–$1s2s\,{}^3S$ to the intercombination line $1s^2\,{}^1S$–$1s2p\,{}^3P$ is derived as a function of collisional excitation rates $C(i \rightarrow j)$, spontaneous transition probabilities $A(i \rightarrow j)$, the photoexcitation rate Φ for the transition $2^3S \rightarrow 2^3P$ and the electron density N_e. Thus

$$\frac{R_0}{R} = \frac{N_e C(2^3S \rightarrow 2^3P) + \Phi}{A(2^3S \rightarrow 1^1S)}(1+F)+1, \tag{1}$$

where

$$F = \frac{C(1^1S \rightarrow 2^3S)}{C(1^1S \rightarrow 2^3P)}, \tag{2}$$

$$R_0 = (1+F)/B+1, \tag{3}$$

and the effective branching ratio B is given by

$$B = \sum_J \frac{g_J}{\Sigma g_J} \frac{A(2^3P_J \rightarrow 1^1S)}{A(2^3P_J \rightarrow 1^1S)+A(2^3P \rightarrow 2^3S)}, \tag{4}$$

summed over the J levels of the 2^3P term. The contribution from the photoexcitation rate Φ is negligible for ions above O vii but is important for C v where the transition is at longer wavelengths and the photospheric flux is high. The value of F was derived from observations in Paper 1 where a value of 0.35 was used. Since this is approximately in accordance with the statistical weight ratio it is acceptable from a theoretical standpoint. It is also supported by recent Coulomb–Born calculations of these transitions (Burgess, Hummer & Tully 1970). In any case, the interpretation is not critically dependent on the value of F used.

It can be seen from equation (1) that at very small N_e, R assumes a limiting value given by

$$R_0 \bigg/ \left(1 + \frac{\Phi(1+F)}{A(2^3\mathrm{S} \rightarrow 1^1\mathrm{S})}\right)$$

which has the value R_0 if $\Phi = 0$. As N_e is increased, R becomes sensitive in the region where $N_e C(2^3\mathrm{S} \rightarrow 2^3\mathrm{P}) \sim A(2^3\mathrm{S} \rightarrow 1^1\mathrm{S})$ and decreases asymptotically to zero. It is possible to derive a value of N_e, say N_e^*, for which R is reduced by 10 % from its value at the low density limit. N_e^* is then the smallest value of N_e which can be detected if the observational accuracy is 10 %. It is given by

$$N_e^* = \frac{0.111}{(1+F)} \frac{A(2^3\mathrm{S} \rightarrow 1^1\mathrm{S})}{C(2^3\mathrm{S} \rightarrow 2^3\mathrm{P})} \left(1 + \frac{\Phi(1+F)}{A(2^3\mathrm{S} \rightarrow 1^1\mathrm{S})}\right). \tag{5}$$

Values of R_0 and N_e^* are given in table 1, while curves showing the variation of R with N_e can be found in Paper 1.

Table 1. Atomic parameters and limiting densities for which the model is sensitive

					(using Griem's values)		(present empirical values)	
ion	Φ/s^{-1}	T_e/K	$C(2^3\mathrm{S} \rightarrow 2^3\mathrm{P})$ / $\mathrm{cm^3\,s^{-1}}$	R_0†	$A(2^3\mathrm{S} \rightarrow 1^1\mathrm{S})$ / $\mathrm{s^{-1}}$	N_e^*† / $\mathrm{cm^{-3}}$	$A(2^3\mathrm{S} \rightarrow 1^1\mathrm{S})$ / $\mathrm{s^{-1}}$	N_e^* / $\mathrm{cm^{-3}}$
C v	2.6^{2}	1.0^{6}	2.80^{-8}	11.0	2.54^{0}	7.5^{6}	3.7^{1}	1.1^{9}
N vi	5.7^{0}	1.3^{6}	1.70^{-8}	5.0	1.00^{1}	4.8^{7}	2.0^{2}	1.3^{9}
O vii	3.7^{-1}	1.9^{6}	1.16^{-8}	3.6	3.22^{1}	2.3^{8}	8.3^{2}	6.0^{9}
Ne ix	—	3.5^{6}	5.82^{-9}	3.0	2.22^{2}	3.1^{9}	9.2^{3}	1.3^{11}
Mg xi	—	5.8^{6}	3.28^{-9}	2.6	1.04^{3}	2.6^{10}	6.3^{4}	1.6^{12}
Al xii	—	7.4^{6}	2.57^{-9}	2.3	2.07^{3}	6.6^{10}	1.5^{5}	4.9^{12}
Si xiii	—	8.9^{6}	2.16^{-9}	2.0	3.93^{3}	1.5^{11}	3.1^{5}	1.2^{13}
S xv	—	1.6^{7}	1.29^{-9}	1.4	1.23^{4}	7.9^{11}	1.3^{6}	8.3^{13}

† These values derived with Φ put equal to zero.
Superscripts indicate the powers of 10 by which the numbers must be multiplied.

3. RECENT OBSERVATIONS

The above analysis has now been applied by Walker & Rugge (1970) to their observations of O vii, Ne ix, Si xiii and S xv from the satellite O v 1–17. They found it necessary to use $F = 0.5$ to fit some of their data observed close in time to solar flares. Observations during flares have also been made from the OSO-V satellite by Neupert & Swartz (1970) and from OSO–IV by Meekins *et al.* (1970), extending up to lines of Fe xxv. It was pointed out in Paper 1 that flare spectra produced during recombination may not adhere strictly to the above model, although they would be expected to give a very similar dependence of R on N_e. Since flare spectra will be excluded from consideration in this paper, it is permissible to carry through all analyses with a consistent value of $F = 0.35$.

A recent rocket experiment, SL 801, from this laboratory, recorded the solar soft X-ray spectrum using a photographic grating spectrograph. A portion of the spectrum is reproduced in figure 1, showing helium-like spectra of C v, N vi and O vii. Since the N vi intercombination line is too weak to record, only an upper limit can be found for its intensity. Details of this spectrum will be published separately (Freeman & Jones 1970). The values of the intensity ratio R are 1.0, $\geqslant 1.9$ and 1.9 for C v, N vi and O vii respectively.

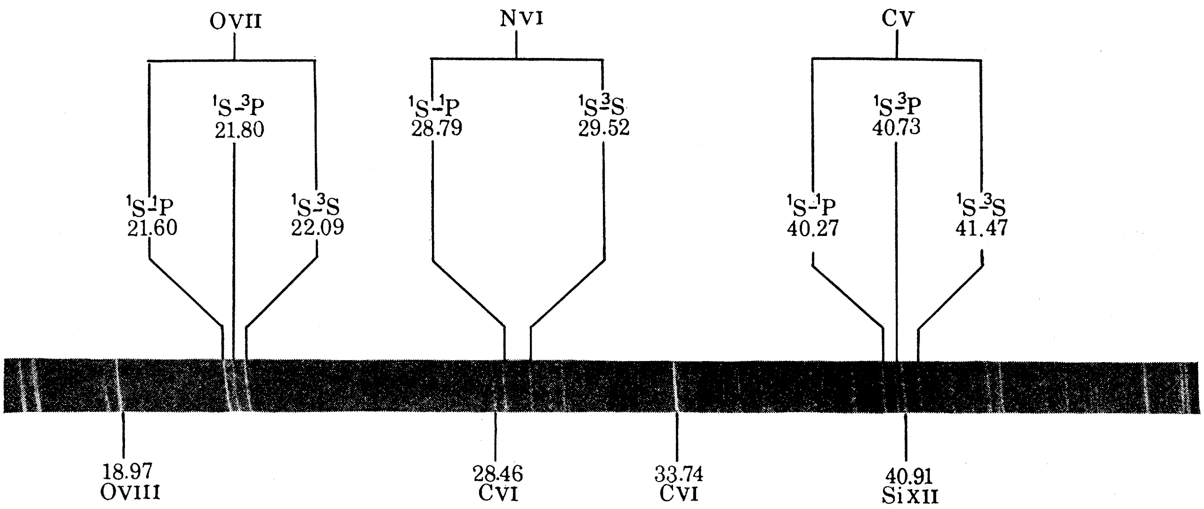

FIGURE 1. Part of the soft X-ray spectrum from Skylark SL 801, flown by the Astrophysics Research Unit on 20 November 1969. Helium-like spectra from C v, N vi and O vii are indicated. (The wavelengths are in ångstöms.)

TABLE 2. ELECTRON DENSITIES DERIVED USING PREVIOUS AND PRESENT A VALUES

		N_e cm^{-3}		
ion	R	(Griem's A values)†	(present A values)	observation
C v	1.0	6.5^{8}	1.3^{9}	SL 801 (Freeman & Jones 1970)
N vi	1.9	6.6^{8}	1.4^{10}	SL 801 (Freeman & Jones 1970)
O vii	2.3	1.1^{9}	3.0^{10}	Fritz *et al.* (1967)
	1.9	1.8^{9}	4.7^{10}	Fritz *et al.* (1967)
	3.6	2.3^{8}	6.0^{9}	Rugge & Walker (1968)
	1.5	3.0^{9}	7.8^{10}	SL 408 (Freeman & Jones 1970)
	1.9	1.8^{9}	4.7^{10}	SL 801 (Freeman & Jones 1970)
	3.7	2.3^{8}	6.0^{9}	Walker & Rugge (1970)
Ne ix	3.0	3.1^{9}	1.3^{11}	Fritz *et al.* (1967)
	2.2	1.0^{10}	4.2^{11}	Fritz *et al.* (1967)
	1.4 ± 0.5	3.1^{10}	1.3^{12}	Evans & Pounds (see Paper 1)
	2.1	1.2^{10}	4.9^{11}	Walker & Rugge (1970)
Mg xi	1.85	9.4^{10}	5.7^{12}	Evans & Pounds (see Paper 1)
	2.4	2.6^{10}	1.6^{12}	Evans & Pounds (see Paper 1)
Al xii	1.85	1.5^{11}	1.1^{13}	Walker & Rugge (1970)
Si xiii	1.6	1.5^{11}	1.2^{13}	Evans & Pounds (see Paper 1)
	1.75	1.5^{11}	1.2^{13}	Walker & Rugge (1970)
S xv	1.0	2.7^{12}	3^{14}	Walker & Rugge (1970)

† These values derived with Φ put equal to zero.
(Superscripts indicate the powers of 10 by which the numbers must be multiplied).

The observations discussed in Paper 1, together with some of the above newer data, have been analysed using Griem's forbidden-line A values. The results are listed in table 2, and plotted in figure 2 in the form of N_e against the electron temperature T_e, each ion being plotted at the temperature at which its emission would be a maximum in equilibrium conditions. As explained in Paper 1, the observed ratio in C v is not acceptable when we take account of the high value of ϕ of $260\,\mathrm{s}^{-1}$. In order to interpret the C v ratios, it is necessary to assume that $\phi = 0$ and the value plotted is derived on this basis. This discrepancy must be regarded as a serious problem in the use of Griem's A values. The dashed line in figure 2 indicates the values of N_e^* listed in table 1. Since these are the smallest values of density that can be distinguished from zero density, it should be noted that points plotted on or near this line do not give a definite density,

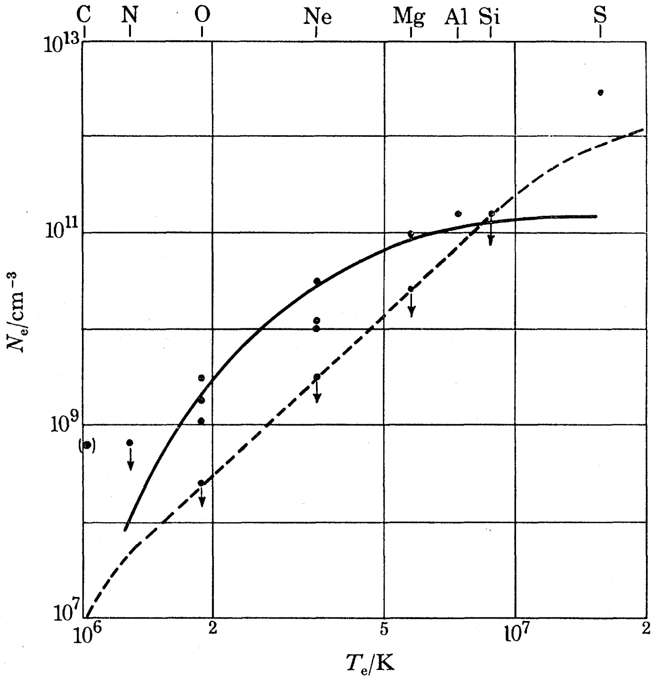

FIGURE 2. Derived values of electron density plotted against the electron temperature at which each ion has its peak emission, using Griem's A values. The dashed curve indicates the low density limit for the method. The observed line ratios would be insensitive to electron density for densities on or below this curve.

but only the inequality $N_e \leqslant N_e^*$. The apparently significant value of N_e for S xv depends on a single uncorroborated observation, so that some doubt may be felt concerning the evidence for density being above N_e^* for this ion. The full line has been drawn through the mean densities indicated for each ion. Since the higher ions clearly arise from active regions, which will vary in their conditions, the scatter of the points about this line is to be expected.

In interpreting figure 2, it must be realized that if the departure of the measured ratios R from the values of R_0 given in table 1 are not considered significant, then all densities can be less than N_e^*, and any conditions below the dashed curve would be allowable. However, the deviations both from R_0 and between the various values of R for each ion are considered by the experimenters to be significant, and in many cases amount to a factor of about 2 in relative intensities, so that real densities greater than N_e^* are indicated.

4. THE FORBIDDEN TRANSITION PROBABILITY

Several workers (R. H. Garstang; I. P. Grant; S. Woosley; private communications) have expressed doubts concerning the values for $A(2^3S \rightarrow 1^1S)$, and in particular have drawn our attention to a possible error in the Z scaling derived by Griem. Breit & Teller (1940) give the result of a Dirac theory calculation of $A(2^2S \rightarrow 1^2S)$ in hydrogen. From Breit & Teller and also from Bethe & Salpeter (1957) we find that this should be scaled as Z^{10} and not Z^8 as used by Griem. Thus we have for hydrogenic ions

$$A = 5\times10^{-6}Z^{10}\ \mathrm{s}^{-1}. \tag{6}$$

Approximating this for helium-like ions $2^3\mathrm{S}\rightarrow1^1\mathrm{S}$:

$$A = \tfrac{2}{3}\times5\times10^{-6}Z_{\mathrm{eff}}^{10}\ \mathrm{s}^{-1}, \tag{7}$$

where the effective charge Z_{eff} can be expressed in terms of the actual wavelength of the transition:

$$Z_{\mathrm{eff}}^2 = 1215/\lambda \quad (\lambda \text{ in Å}). \tag{8}$$

So that

$$A = 8.8\times10^9/\lambda^5\ \mathrm{s}^{-1}. \tag{9}$$

We note that for C v, the observed ratio R of 1.0 departs considerably from 11.0 for R_0. Furthermore, if we also use the calculated value of $\phi = 260\,\mathrm{s}^{-1}$, it is possible to derive a value for $A(2^3\mathrm{S}\rightarrow1^1\mathrm{S})$ which is quite insensitive to the value of N_e chosen. Taking $N_e \approx 10^9\,\mathrm{cm}^{-3}$ gives a value for A of $37\,\mathrm{s}^{-1}$. If we accept the scaling of equation (9) but modify the constant to give this value we get

$$A = 4.4\times10^9/\lambda^5\,\mathrm{s}^{-1}. \tag{10}$$

It should be realized that equation (6) on which the Z scaling is based is a one-electron formalism, and no account has been taken of interaction between the two electrons. Additional terms due to this are unlikely to be much larger (equation (10) already leads to very high densities), and we therefore believe that this derived value for A is likely to be good to a factor of about 2.

5. INTERPRETATION

With these new derived values, it is possible to recompute N_e, now taking full account of the photoexcitation rate ϕ. This leads to the final columns in tables 1 and 2. The results are plotted in figure 3. Again the values of N_e^* are shown as a dashed curve, the upward turn at C v being due to the effect of Φ. N vi also gives only a maximum value. However, from O vii to Al xii the values of N_e appear to be significantly above N_e^*. The densities now derived are even higher than before, ranging from $3\times10^{10}\,\mathrm{cm}^{-3}$ for O vii up to almost $10^{13}\,\mathrm{cm}^{-3}$. These values are very large, but in view of the lack of other determinations are not impossible for the active regions in which they occur. The magnetic field required to contain a plasma of $5\times10^6\,\mathrm{K}$ at $10^{13}\,\mathrm{cm}^{-3}$ is ~ 40 mT (400 G), which could be available in such regions.

We can also derive an emission measure (N_e^2V) from the absolute intensities of the lines observed on the SL 801 spectrum. This is a factor of 5 lower for active region lines formed at about $5\times10^6\,\mathrm{K}$ than for quiet coronal lines formed at $1.5\times10^6\,\mathrm{K}$. If the density ratio between active and quiet regions is about 10^4, and taking the volume of the corona as $2\times10^{32}\,\mathrm{cm}^3$, this leads to a volume for the hotter emission of about $5\times10^{23}\,\mathrm{cm}^3$, equivalent to a filament of length 30000 km and diameter 150 km. We know from high spatial resolution photographs by Vaiana *et al.* (1968) that the active region radiation comes from very small regions. If the actual volume is as small as derived here, it must consist of unresolved condensations within the observed emitting regions.

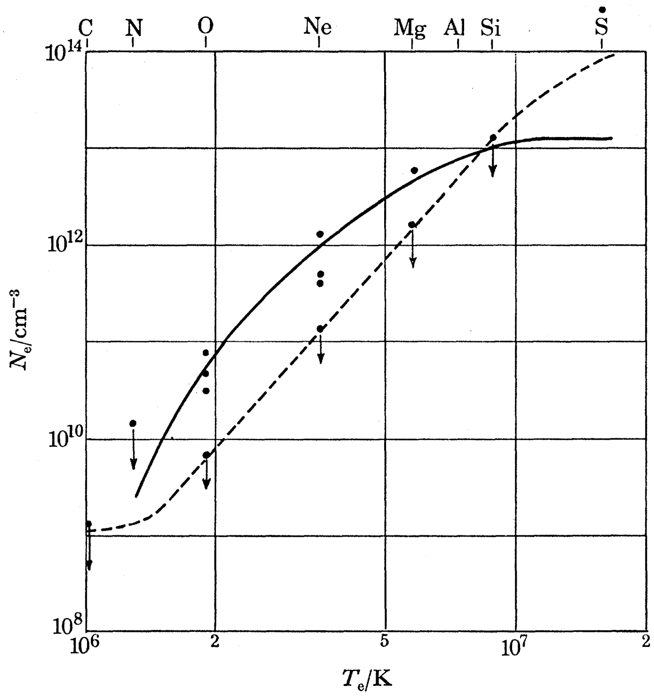

FIGURE 3. The same data as in figure 2 with the densities derived using the present semi-empirical A values.

6. CONCLUSIONS

The theory proposed in Paper 1 has been applied to a wide range of observations, including a number of recent spectra. Using the forbidden line transition probabilities calculated by Griem, active region densities up to 10^{11} cm^{-3} are derived, but it is not possible to explain the C v observations when full account is taken of the $2^3S \rightarrow 2^3P$ photoexcitation rate.

New values for the forbidden line transition probabilities are proposed. These are derived in a semi-empirical manner, using a Z scaling based on hydrogenic Dirac theory results, together with an absolute value from the C v observations. When the analysis is carried through with these new values, even higher active region densities, up to 10^{13} cm^{-3}, result. Correlation of these densities with the observed emission measure leads to a small emitting volume of about 5×10^{23} cm^3.

We are pleased to acknowledge helpful discussions with Professors R. H. Garstang and H. R. Griem.

Note added in proof (25 *January* 1971)

Since submitting this manuscript, Dr G. W. F. Drake (*Phys. Rev.* A., in press) has carried out more complete calculations of the forbidden line transition probability, taking account of electron–electron interaction. Furthermore, Drs R. W. Schmieder and R. Marrus have succeeded in measuring the decay rate for argon XVII in a beam-foil experiment (*Phys. Rev. Lett.* **25**, 1245 (1970)). Both results support the above derived semi-empirical values to within 25 %.

Discussion

K. A. Pounds (*University of Leicester*). While one might be able to accept the high densities in the case of the higher ions, is not the problem more severe for C v and O vii?

A. H. Gabriel. There is no indication from the measurements of high densities for C v. However, the high densities derived for O vii are certainly difficult to understand.

M. J. Seaton (*University College London*). What effect do the high densities have on the ionization equilibrium?

C. Jordan. The di-electronic recombination rates from hydrogen-like and helium-like ions are not as large as from more complex ions. A reduction of the di-electronic recombination rates due to density effects would not substantially alter the ionization equilibrium for the helium-like ions.

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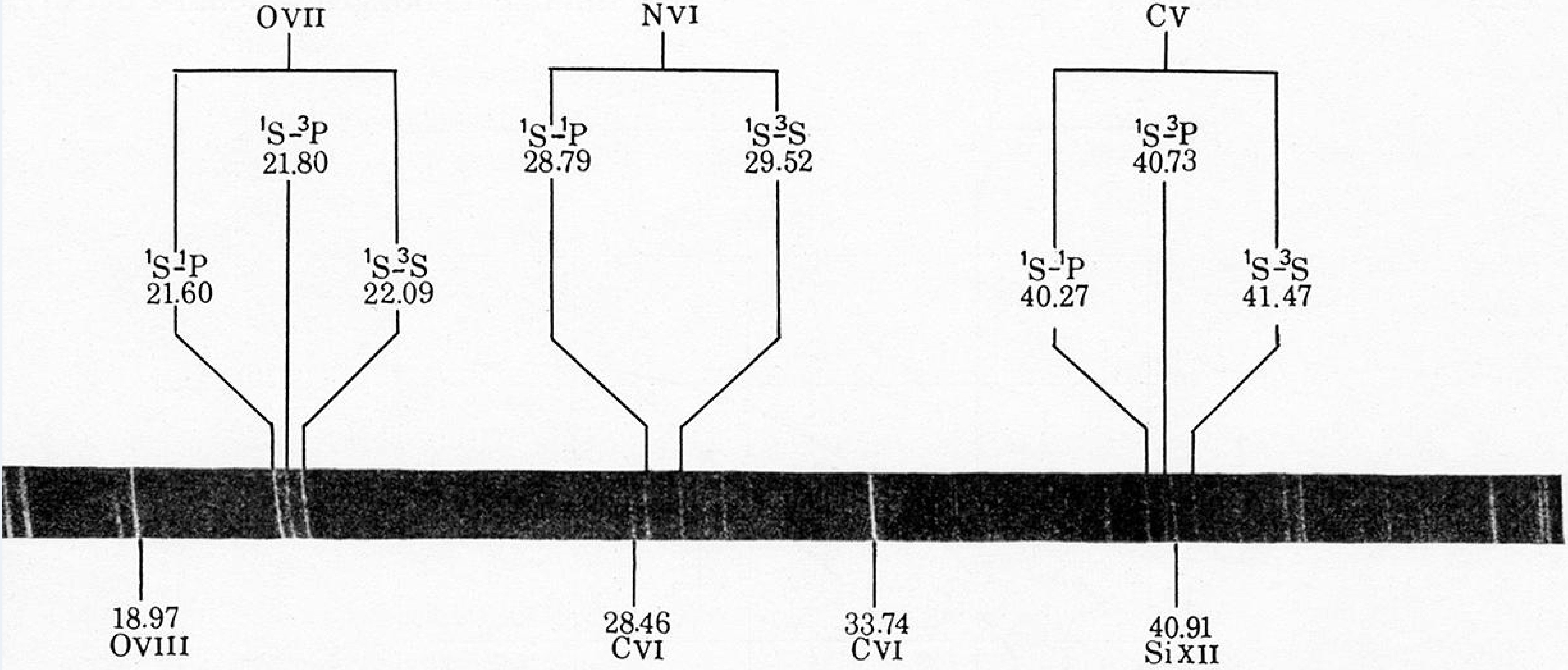

FIGURE 1. Part of the soft X-ray spectrum from Skylark SL 801, flown by the Astrophysics Research Unit on 20 November 1969. Helium-like spectra from C v, N vi and O vii are indicated. (The wavelengths are in ångstöms.)